# Predictive Discordancy Tests For Exponential Observations

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## Summary

Using a Bayesian approach, unconditional and conditional predictive testing procedures are proposed for the detection of discordant observations. These tests may be adjusted to take account of diagnostics which search the data for potentially discordant observations. The technical details are given for the translated exponential distribution in the presence of censored observations.

Some key words: Bayesian approach, Conditional predictive ordinate (CPO), Conditional predictive discordancy (CPD) tests, Discordancy indices, Discordancy tests, Predictive influence functions, Translated exponential distribution, Unconditional predictive discordancy (UPD) tests.

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## 1. Introduction.

In this paper we devise a Bayesian predictive approach for significance testing of potentially discordant observations (or outliers) in situations where the sampling is assumed to be from a translated exponential population. As in all pure significance tests it is presumed that the possible alternatives are not easily specified. In potential discordancy situations a small enough P-value will alert the statistician or investigator to give careful attention to the observation(s) in question and make a determination of how it (they) should be used in the analysis of the data. This might entail inclusion, exclusion or modification of the observation(s) or even in some cases a revision of the sampling model.

In section 2 we shall discuss in general predictive significance tests when an observation has been indentified because of the intrusion of some untoward event or taken under suspicious circumstances that may or may not have effected its value. To handle this situation, Unconditional Predictive Discorancy (UPD) and Conditional Predictive Discordancy (CPD) tests are introduced.

Section 3 is devoted to situations where the data is ransacked by means of diagnostics or discordancy indices that rank abbreviations as to their relative discordance such that the most discordant is a candidate for testing. The previous UPD and CPD tests are then modified to take account of the ransacking. The next few sections focus on applying these ideas and developing the requisite test procedures for translated exponential populations.

## 2. Testing an observable that was identifiable for reasons other than its value.

We consider the situation where $Y_1, \ldots, Y_N$ are independently and identically distributed with common distribution function $F_Y(y|\theta)$, and an assumed prior density $g(\theta)$. Hence one can compute the predictive distribution for a future value (or set of values) Z by calculating

$$F_Z(z|y^{(N)}) = E_\Theta F_Y(z|\theta)$$

where $y^{(N)} = (y_1, \ldots, y_N)$ is the observed set of values of $Y^{(N)} = (Y_1, \ldots, Y_N)$, and the expectation is over the posterior distribution of $\Theta$.

On obtaining an experimental value for $Y_i$, say, it was noted that some untoward event occurred that could have possibly influenced the value of the observation. The experimenter might want to decide whether to include the observed value $y_i$. By calculating the predictive distribution of Z, based on all the observations except for $y_i$ and denoted $y_{(i)}$,

$$F_Z(y_i|y_{(i)}) = \int F_Z(y_i|\theta)dP(\theta|y_{(i)})$$

where $P(\theta|y_{(i)})$ is the posterior distribution of $\theta$ given $Y_{(i)} = y_{(i)}$, an assessment of the discordancy of $Y_i$ with $Y_{(i)}$ can be made. A P-value, Geisser (1980), can be calculated where

$$P_i = Pr[Z \in R_i|y_{(i)}] \tag{1}$$

for some suitably defined region $R_i$ using the conditional predictive distribution of Z given $y_{(i)}$.

This procedure assumes that only $y_i$ is suspect and that the rest concord with the model, allowing one to condition on $y_{(i)}$. If a test is required which does not depend on the assumption that $y_{(i)}$ is concordant then one could calculate the marginal distribution of $Y_i$,

$$F(y) = \int F(y|\theta)dG(\theta)$$

where $G(\theta)$ is the prior distribution for $\theta$. A computation analgous to (1) can be made using the unconditional distribution of $Y_i$, namely

$$P_i = Pr[Y_i \epsilon R]. \tag{2}$$

This has the advantage of being completely independent of any assumption on $Y_{(i)}$, but can only be used with a proper prior distribution. Of course when the assumption that $Y_{(i)}$ concords with the model can confidently be made, the more sensitive conditional test, previously described, is available and uses the additional information. Further this test exists for certain useful improper prior distributions as well.

Often after seeing the data a statistician may wish to check for potential discordancies or outliers before proceeding with an analysis. In the predictive framework he may ransack the data by calculating any of several diagnostic discordancy indices. We shall briefly review three of them.

Johnson and Geisser (1982, 1983) proposed predictive influence functions (PIF) using the Kullback-Leibler numbers between the predictive distribution of a future value based on $y_{(i)}$ and $y^{(N)}$ for determining influential observations. This can also be used in the i.i.d. case to yield an ordering of the discordancy of the observations, i.e. the observation yielding the largest Kullback-Leibler number is the prime candidate for a discordancy test. Thus the PIF,

$$I_i = E[\ln f(Z|y_{(i)}) - \ln f(Z|y^{(N)})]$$

where the expectation is taken over the predictive distribution $F_Z(z|y_{(i)})$, i.e. with $y_i$ deleted can serve as a discordancy index.

Another such diagnostic or index, Geisser (1980,1985),

$$d_i = f(y_i|y_{(i)})$$

called the Conditional Predictive Ordinate (CPO) ranks the discordancy of observations--the smaller the value of $d_i$ the more discrepant is $y_i$ from $y_{(i)}$. Another index is either one of the previous predictive tests, (1) or (2) of section 1, which can be used to rank discordancy.

3. Predictive Discordancy Tests based on Diagnostics.

Suppose that the diagnostic, say H, chooses $y_C$ as potentially most discordant where the observed value of $H(y_C) = h_C$

$$h_C \succ h_i \quad \text{for all } i \neq C$$

where $\succ$ stands for "more discordant." One can check for discordance by calculating a significance level from $h_C$, the observed value of $H_C$,

$$Pr[H_C \succ h_C | C]$$

where $F_{H_C}(h_C)$ is calculated from $F(y^{(N)})$ under choice C i.e. that $h_C$ was the observed value of the most discrepant diagnostic.

Hence we have modified the UPD test approach to take account of the diagnostic ransacking.

A second approach is to modify the CPD test by adjusting for the ransacking. One way is to calculate

$$P_C = Pr[H(Z) \succ h_C | H(Z) \succ h_{C-1}; y_{(C)}]$$

where H(Z) is the conditional distribution of the diagnostic H calculated from the predictive conditional distribution

$$f(z|y_{(C)}).$$

and $h_{C-1} = H(y_{C-1})$ for $y_{C-1}$ the second most discrepant diagnostic. For example when the diagnostic is merely a monotone increasing function of y, then

$$P_C = Pr[Z > y_C | Z > y_{C-1}; y_{(C)}].$$

We shall use these approaches to devise discordancy tests for translated exponential populations.

## 4. Translated Exponential Distribution.

We briefly review a Bayesian approach to translated exponential observables. Let $Y_1, \ldots, Y_N$ be a random sample from

$$f(y|\alpha,\gamma) = \alpha e^{-\alpha(y-\gamma)} \qquad y > \gamma,\ \alpha > 0.$$

Let $y_1, \ldots, y_d$ represent fully observed values and $Y_{d+1}, \ldots, Y_N$ be censored at $y_{d+1}, \ldots, y_N$ respectively.

Let $m = \min(y_1, \ldots, y_d)$ and for reasons previously discussed, Geisser (1984), we assume that

$$m < \min(y_{d+1}, \ldots, y_N).$$

Let the conjugate prior density be

$$g(\gamma,\alpha) = g(\gamma|\alpha)g(\alpha)$$

where

$$g(\gamma|\alpha) = N_0\alpha e^{\alpha N_0(\gamma-m_0)}, \qquad \gamma < m_0$$

and

$$g(\alpha) \propto \alpha^{d_0-2} e^{-\alpha N_0(\bar{y}_0-m_0)} \qquad \alpha > 0,\ \bar{y}_0 > m_0$$

where $1 < d_0 \leq N_0$. Then Geisser (1984) obtains for the posterior densities

$$p(\gamma|\alpha) \propto e^{\alpha N^*(\gamma-m^*)} \qquad \gamma < m^*$$

$$p(\alpha) \propto \alpha^{d^*-2} e^{-\alpha N^*(\bar{y}^*-m^*)} \qquad \bar{y}^* > m^*,\ \alpha > 0$$

for $1 < d^* \le N^*$, $d^* = d_0 + d$, $N^* = N_0 + N$, $m^* = \min(m_0, m)$,

$\bar{y}^* = (N_0+N)^{-1}(N_0\bar{y}_0+N\bar{y})$ and $N\bar{y} = \sum_i y_i$.

Note that for the noninformative prior

$$g(\gamma,\alpha) \propto \alpha^{-1},$$

$m^* \dashrightarrow m$, $\bar{y}^* \dashrightarrow \bar{y}$, $d^* \dashrightarrow d$, $N^* \dashrightarrow N$.

The predictive distribution of a future observable Z, in the non-informative case is

$$F(z) = \begin{cases} \frac{1}{N+1}\left(\frac{\bar{y}-m}{\bar{y}-z}\right)^{d-1} & z \le m \\ 1 - \frac{N^d(\bar{y}-m)^{d-1}}{(N+1)[z-m+N(\bar{y}-m)]^{d-1}} & z > m. \end{cases}$$

Geisser (1984). Note that we need only add stars to m, $\bar{y}$, d or N to recoup the case for the conjugate prior.

5. Predictive Discordancy Indices.

The predictive distribution of a future observable Z as given in section 4 can be used to define several different conditional predictive discordancy indices. We shall restrict our discussion to the non-informative case for the time being. Because we are dealing with independent and identically distributed observables the Kullback-Leibler predictive influence function here can serve as one indicator of discordancy. The actual calculation of this index is quite tedious but it can be shown that the potentially most influential observation will generally be either be m or M, the largest among all values. The most influential, of course, is also in this case the prime candidate for discordancy.

The Conditional Predictive Ordinate (CPO) is considerably easier to calculate. For an uncensored value $y_i \neq m$ the CPO

$$d_i = f_{(i)}(y_i | y_{(i)}) = \frac{(d-2)(N-1)^{d-1}(\bar{y}_{(i)}-m)^{d-2}}{N[N(\bar{y}-m)]^{d-1}}$$

which clearly shows that the largest $y_i \neq m$, $i = 1,\ldots, d$ has the smallest CPO. For $y_i = m$ and $m_2$ smaller than any uncensored value we obtain

$$d_m = \frac{d-2}{N} \frac{(\bar{y}_{(m)}-m_2)^{d-2}}{(\bar{y}_{(m)}-m)^{d-1}}$$

and

$$\min(d_i) = \min(d_m, d_{M_u})$$

where $M_u$ is the largest uncensored observation. For the censored observations $i = d+1,\ldots, N$

$$d_i = \frac{(d-1)(N-1)^d}{N} \frac{(\bar{y}_{(i)}-m)^{d-1}}{(N(\bar{y}-m))^d}$$

and if the largest uncensored value is about the same as the largest censored value then its CPO will be smaller. Basically this diagnostic will choose either the largest value or the smallest value. Clearly the tail area diagnostic $P_i$ will give somewhat similar results. The possibly minor differences in the three discordancy indices depend on the unusual shape of the predictive density function, but they all tend to focus on the extreme values as potential candidates for discordancy. Some adjustments may need to be made when the proper prior is used, as the indices will also depend on the hyperparameters. However when a non-extreme value turns out to yield the most discordant index, it is usually unncessary to test it for discordance.

Clearly diagnostics can also be based either on unconditional predictive ordinates or their "tail area" P-values and used in a similar fashion.

## 6. Unconditional Discordancy Tests.

We shall present the unconditional tests for discordancy first for the largest observation and then for the smallest assuming these are the only candidates. For the largest observation, say M, calculate

$$P_M = 1 - \int F(M|\theta)g(\theta|\Upsilon,\alpha)d\theta$$

where $F(M|\theta)$ is the distribution of the maximum conditional on $\theta$, and obtain

$$P_M = \sum_{j=1}^{N} \binom{N}{j}(-1)^{j+1}\left(\frac{N_0}{N_0+j}\right)\left[\frac{N_0(\bar{y}_0-m_0)}{N_0(\bar{y}_0-m_0)+(M-m_0)j}\right]^{d_0-1}$$

Since this represents the probability that the maximum is at least as large as is observed value the result is appropriate for the maximum observation whether fully observed or censored.

Similarly, for the smallest observation m, we obtain

$$P_m = 1 - \left(\frac{N_0}{N_0+N}\right) \frac{[N_0(\bar{y}_0-m_0)]^{d_0-1}}{[N(m-m_0)+N_0(y_0-m_0)]^{d_0-1}} \qquad \text{for } m \geq m_0$$

$$P_m = \frac{N}{N_0+N}\left(\frac{\bar{y}_0-m_0}{\bar{y}_0-m}\right)^{d_0-1} \qquad m \leq m_0$$

Of course the above tests exist only for the proper prior distribution.

7. Conditional Predictive Discordancy Tests.

In the light of the previous remarks we now present CPD tests for the extreme values. For a conditional predictive test for the discordancy of the smallest observation we propose the significance level calculated as

$$P_m = \Pr[Z \leq m | Z \leq m_2, y_{(m)}] \ .$$

For the proper prior we obtain

$$P_m = \begin{cases} A(m)/A(m_2) & m_0 \leq m \\ B(m)/A(m_2) & m \leq m_0 \leq m_2 \\ B(m)/B(m_2) & m \leq m_2 \leq m_0 \end{cases}$$

where

$$A(z) = 1 - \frac{N^*-1}{N^*}\left(\frac{(N^*-1)(\bar{y}_{(m)}-m_0)}{(N^*-1)(\bar{y}^*_{(m)}-m_0) + z-m_0}\right)^{d^*-2}$$

$$B(z) = \frac{1}{N^*}\left(\frac{\bar{y}^*_{(m)} - m_0}{\bar{y}^*_{(m)} - z}\right)^{d^*-2}$$

The non-informative prior, however, yields the simple form

$$P_m = \left(\frac{\bar{y}_{(m)} - m_2}{\bar{y}_{(m)} - m}\right)^{d-2} .$$

We illustrate this with some data from Kabe (1970) on lifetimes in hours of 5 pieces of a metal material. The values are 525, 603, 621, 648, 663. In this case we calculate for the non-informative case,

$$P_m = .023.$$

Kabe, using the frequentist approach and Dixon's (1950, 1951) test statistic $T_m$ with realized value

$$t = \frac{m_2 - m}{M - m} = .565,$$

calculates an exact significance level to be $\alpha = 0.027$ (this appears to be erroneous with the correct result being .0164).

It is of interest to note that for $d = N$, it can easily be shown that the conditional frequency calculation

$$\Pr[T_m > t \mid U = u] = (1-ut)^{N-2} = P_m, \quad \text{for } t \leq u^{-1}$$

where

$$u = \frac{M-m}{N(\bar{y}-m)},$$

and t the realized value of $T_m$.

For a CPD test for the largest observation we suggest, in the non-informative case,

$$P_M = \Pr[Z \geq M | Z > M_2, y_{(M)}] = \left[\frac{M_2-m+(N-1)(\bar{y}_{(M)}-m)}{M-m+(N-1)(\bar{y}_{(M)}-m)}\right]^c$$

where $M_2$ is the second largest observation and $c = d-2$ or $d-1$ depending on whether M was an uncensored or censored observation. (For the conjugate prior it is only necessary to star N, m, d and $\bar{y}_{(m)}$.) For the sake of comparing calculations only we present some data representing an analysis of phosphorous as a component of carbon steel, used by Likes (1966) to test for the largest value as an outlier. The data, in $10^6$ multiples, are 4, 6.33, 7, 7, 9, 9.33, 25. For this data we obtain

$$P_M = .048$$

and compare this with the $\alpha$ level of the usual frequentist test statistic $T_M$ with realized value

$$t = \frac{M-M_2}{M-m}$$

as given by Likes, where $\alpha = (N-1)(N-2)B\left(\frac{2-t}{1-t}, N-2\right)$

and B(.,.) is the beta function. For this particular example, $\alpha = 0.05$.

If $\gamma$ is known one can calculate from the predictive distribution of Z

$$P_M = \left[\frac{M_2 - \gamma + (N-1)(\bar{y}_{(M)}-\gamma)}{M - \gamma + (N-1)(\bar{y}_{(M)}-\gamma)}\right]^{c+1}$$

where c is defined as for the case when $\gamma$ is unknown.

Again for d = N, a conditional frequency calculation,

$$\Pr[T_M > t | U=u] = (1-tu)^{N-2} = P_M, \quad \text{for } t \leq u^{-1}$$

where t is the realized value of $T_M$, is available.

To illustrate this with censored data we consider the data set in the following table from one of two groups reported in Pike (1966).

TABLE 1
Days (Y) to Vaginal Cancer Mortality in Rats

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 143, | 164, | 188, | 190, | 192, | 206, | 209, | 213, | 216 |
| 220, | 227, | 230, | 234, | 246, | 265, | 304, | 216* | 244* |

*censored

He assumes that $U = (Y-100)^3$ is exponentially distributed. On this basis we shall test for the discordance of the largest observation M = 304. If there had been some initial suspicion regarding this particular rat the significance test which yields P = .015 would have confirmed the concern. However if the value was chosen merely on the basis that it was the largest, and since $M_2 = 265$, we can calulate

$$P_M = .156$$

which is larger than the initial P-value by an order of magnitude and is not a particularly surprising result if the exponential model is reasonably adequate as claimed.

## 8. Tests For Combinations of Largest and Smallest.

In order to derive CPD tests for 2 observations at a time, i.e. combinations of the smallest and largest, we need the joint predictive distribution of two future observations given $y^{(N)}$. We will consider the unstarred case in what follows. The preliminary relevant calculations are:

$$\Pr[Z_1 \le z_1, Z_2 \le z_2 | y^{(N)}] = \left(\frac{\bar{y}-m}{\bar{y}-v}\right)^{d-1} + \frac{N}{N+2}\left[\frac{N(\bar{y}-m)}{N\bar{y}+z_1+z_2-(N+2)v}\right]^{d-1}$$
$$- \frac{N}{N+1}\left[\frac{N(\bar{y}-m)}{N(\bar{y}-v)+z_2-v}\right]^{d-1} - \frac{N}{N+1}\left[\frac{N(\bar{y}-m)}{N(\bar{y}-v)+z_1-v}\right]^{d-1}$$

for $v = \min(z_1, z_2)$ and $\max(z_1, z_2) \le m$,

$$\Pr[Z_1 \le z_1, Z_2 > z_2 | y^{(N)}] = \frac{N}{(N+1)(N+2)}\left[\frac{N(\bar{y}-m)}{N(\bar{y}-z_1)+z_2-z_1}\right]^{d-1}$$

for $z_1 \le m \le z_2$,

$$\Pr[Z_1 > z_1, Z_2 > z_2 | y^{(N)}] = \frac{N}{N+2}\left[\frac{N(\bar{y}-m)}{N(\bar{y}-m)+z_1+z_2-2m}\right]^{d-1}$$

for $\min(z_1, z_2) \ge m$.

For a joint discordancy test of the smallest and largest $(m,M)$ we suggest, using the calculation from above and the fact that $Z_1$ and $Z_2$ are exchangeable, the significance level

$$\begin{aligned} P_{m,M} &= \Pr[Z_1 \le m,\ Z_2 > M | Z_1 \le m_2,\ Z_2 > M_2,\ y_{(m,M)}] \\ &= \left[\frac{(N-2)(\bar{y}_{(M,m)}-m_2) + M_2 - m_2}{(N-2)(\bar{y}_{(M,m)}-m) + M - m}\right]^c \\ &= [1-u(t_M+(N-1)t_m)]^c \end{aligned}$$

where $c = d-2$ if $M$ is censored, and $d-3$ if $M$ is uncensored and $\bar{y}_{(M,m)}$ is the mean of all the observations excluding m and M.

To illustrate this we use the data on the 5 test pieces of metal that we used for testing the discordancy of the minimum. We obtain

$$P_{m,M} = .062.$$

The usual frequentist test depends on the statistic T with observed value

$$t = \frac{M_2 - m_2}{M - m}$$

and $\alpha = P(T \le t)$ where

$$\alpha = 1 - (N-1)!(1-t)^2 \sum_{j=1}^{N-3} \frac{(-1)^{j+1} j[N-1-(N-j-2)t]^{-1}}{(j+1)!(N-3-j)!(1+jt)!},$$

a result given by Kabe (1970) but with a typographical error which was corrected by Barnett and Lewis (1978). For this case $\alpha = .071$.

For $d = N$, it can easily be shown that the conditional frequency calculation

$$\Pr[T_M + (N-1)T_m > t_M + (N-1)t_m | u] = P_{m,M}$$

where $t_M$ and $t_m$ are the realized values of $T_M$ and $T_m$ respectively and

$$0 \le t_M + (N-1)t_m \le u^{-1}.$$

For the two smallest $(m, m_2)$ where $m_3$ is the third smallest and assuming $m_3 \le \min(y_{d+1}, \ldots, y_N)$ it seems plausible to calculate

$$P_{m,m_2} = \Pr[Z_1 \le m, m \le Z_2 \le m_2 | Z_1 \le Z_2 \le m_3, y_{(m,m_2)}]$$

$$= (N-2)\left\{\left[\frac{\bar{y}_{(m,m_2)}-m_3}{\bar{y}_{(m,m_2)}-m}\right]^{d-3} - \left[\frac{(N-2)(\bar{y}_{(m,m_2)}-m_3)}{N(\bar{y}-m)}\right]^{d-3}\right\}$$

for $N \ge d > 3$.

For the two largest $(M,M_2)$, similarly we may calculate

$$P_{M,M_2} = \Pr[Z_1 > M, M_2 < Z_2 \le M | Z_1 > Z_2 \ge M_3, y_{(M,M_2)}]$$

$$= 2\frac{\{\Pr[Z_1 > M, Z_2 > M_2] - \Pr[Z_1 > M, Z_2 > M\ ]\}}{\Pr[Z_1 > M_3, Z_2 > M_3]},$$

where $M_3$ is the third largest observation. Then for

$$c = \begin{cases} d-1 & \text{if } M \text{ and } M_2 \text{ are censored} \\ d-2 & \text{if one of } M \text{ or } M_2 \text{ is censored} \\ d-3 & \text{if } M \text{ and } M_2 \text{ are uncensored,} \end{cases}$$

$$P_{M,M_2} = 2[(N-2)(\bar{y}_{(M,M_2)}-m) + 2(M_3-m)]^c$$
$$\times\left\{[(N-2)(\bar{y}_{(M,M_2)}-m)+M+M_2-2m]^{-c} - [(N-2)(\bar{y}_{(M,M_2}-m)+2(M-m)\right\}^{-c}.$$

For the case where $\gamma$ is known $P_{M,M_2}$ is calculated as above but with $\gamma$ substituted for m and c+1 for c.

It is of interest to point out that plausible alternative regions can be used for testing the two largest or the two smallest that have frequentist analogues when d = N. It is not difficult to show that defining

$$P'_{M,M_2} = \frac{\Pr[Z_1 > M,\ Z_2 > M_2 | y_{(M,M_2)}]}{\Pr[Z_1 > M_3,\ Z_2 > M_3 | y_{(M,M_2)}]}$$

will result in

$$P'_{M,M_2} = (1-ur)^c$$

where c is defined as before in the censored case and

$$r = \frac{(M-M_2) + 2(M_2-M_3)}{M-m} .$$

Further for $d = N$, the frequency calcuation for the random variable R observed as r is

$$\Pr[R \geq r|u] = P'_{M,M_2} .$$

A similar calculation for the two smallest

$$P'_{m,m_2} = \frac{\Pr[Z_1<m,\ Z_2<m_2|y_{(m,m_2)}]}{\Pr[Z_1<m_3,\ Z_2<m_3|y_{(m,m_2)}]} ,$$

results in

$$P'_{m,m_2} = (1-us)^{d-3}$$

where

$$s = \frac{(N-1)(m_2-m) + (N-2)(m_3-m_2)}{M-m} .$$

Again for $d = N$ the frequency calculation yields, for S the random variable observed as s,

$$\Pr[S \geq s|u] = P'_{m,m_2}.$$

9. Final Remarks

It is to be recalled that all of the CPD tests can be given in terms of the original proper prior distribution by merely substituting $m^*$, $\bar{y}^*$, $d^*$, $N^*$ for m, $\bar{y}$, d and N respectively. Depending on how the choices are made for investigating discordancy other regions may also be plausible for the calculation of significance. Some of the advantages of the CPD tests given here over the usual frequency discordancy tests are that they can be easily adapted to the presence of prior information and censoring. The comparative advantage of the CPD tests over the UPD tests is that the former can be used with certain useful improper priors and also tend to be much easier to calculate.

It is also to be noted that all of these tests are subjective assessments and, in general, are not frequency based even though under very particular circumstances some of them can be shown to have a frequency analogue.

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